

Tree detection, delineation, and measurement from LiDAR point clouds using RANSAC

Peter Tittmann¹, Sohail Shafii², Bruce Hartsough³, Bernd Hamann⁴

¹Department of Geography, UCD (University of California, Davis),
ptittmann@gmail.com

²Department of Computer Science, UCD, ssshafii@ucdavis.edu

³Department of Biological and Agricultural Engineering,
UCD, brhartsough@ucdavis.edu

⁴Department of Computer Science, UCD, hamann@cs.ucdavis.edu

Abstract

As Light Detection And Ranging (LiDAR) (point) data sets increase in resolution, earth scientists become more interested in detecting and delineating trees using LiDAR. The majority of conventional methods that detect and delineate trees convert point data into gridded surfaces. Unfortunately, this conversion process has the potential to introduce error. We improve a point-based geometric model fitting strategy based on “RANdom Sample Consensus” (RANSAC), known as *StarSac*, and compare the method’s results against field data. The analysis demonstrates that *StarSac* produces similar results to field data, and is a strong alternative to conventional methods.

Keywords: geometric, RANSAC, model fitting

1. Introduction

As the capabilities of aerial Remote Sensing (RS) technologies such as Light Detection And Ranging (LiDAR) increase in precision, the potential to directly measure vegetation characteristics has increased as well. The majority of methods using LiDAR data for individual tree detection and delineation emphasize the conversion of point data into gridded surfaces, and the application of algorithmic tools widely used for terrain surface analysis to identify and delineate individual trees. While these methods have been shown to be effective under a range of circumstances, the interpolation of points to a gridded surface followed by watershed, valley-following, or other such methods have the potential to introduce error from both steps. To reduce the impact of such errors, methods are often parameterized by field data (species, canopy height, etc.) and as a result can be quite accurate in tree detection and delineation. If extensive field data collection is required to parameterize the algorithm, the efficiency of the inventory effort is compromised.

We have developed a “RANdom SAMple Consensus” (RANSAC)-based (Fischler and Bolles 1981) program, henceforth referred to as *StarSac*, which uses a geometric model fitting strategy to identify individual tree crowns directly from point data. Basal area is then calculated using a regression, relating Diameter at Breast Height (DBH, 1.37 m) to total height. We identify the location, and height of all trees within the scene without parameterization from field data using the point data directly and avoid the compounding error problem described above. Field data is used to parameterize the height-DBH regression for estimation of DBH.

1.1 Study site

The study site is the *van Eck forest*, consisting of 879 ha of mixed conifer forestland in Humboldt County, California. The van Eck forest contains a great deal of structural and successional diversity, including riparian areas, selectively logged second growth mixed conifer

stands, and stands which have not been harvested since initial clear cut in the early part of the century. The forest is divided in to four tracts, ranging from 130 ha to 315 ha (See Figure 1). It is a well-stocked second growth forest with an average timber volume of $170 \text{ m}^3 \text{ ha}^{-1}$.



Figure 1: van Eck Study area location and management units.

2. Background

2.1 Detection and delineation

The majority individual tree identification methods using LiDAR data combine the creation of a Digital Surface Model (DSM) and Canopy Height Model (CHM) with Local Maxima (LM) filtering to identify tree locations (Jensen et al. 1987; Kaartinen et al. 2008; Lee et al. 2010; McCombs et al. June 2003; Tesfamichael et al. 2009, 2010). The efficacy of methods relying on LM filtering for tree detection depends on the determination of an analysis window size that reflects the crown area of the trees being identified. Popescu et al. (2002) tested an approach using a variable window defined by stand specific field data. However, this approach is dependent on field observation of crown radii to calibrate the LM window size.

Several methods have been tested to estimate tree crown and bole parameters (radius, bole volume, etc) once tree location has been established. Region growing methods such as the watershed delineation adapted from terrain analysis are common (Hyypä et al. 2001; Schardt et al. 2002; Ziegler et al. 2000). Geometric models of tree crown shapes have also been used to delineate individual trees in Airborne Laser Scanning (ALS) data. Abstract tree crown form was first described by Horn (1971). Shapes were further modified by Pollock (1996). Geometric shapes have been used by others to measure tree crowns from aerial photography and LiDAR (Gong et al. 2002; Holmgren et al. 2003; Pollock 1996; Sheng et al. 2001; Wolf and Heipke 2007, Persson 2001, Persson et al 2002, Andersen et al. 2002, Popescu 2003, Wack et al. 2003, Falkowski et al. 2006, Wolf and Heipke 2007, Heurich 2008, Kaartinen et al. 2008).

2.2 RANSAC, *StarSac*

RANSAC (Fischler and Bolles 1981) is a paradigm for fitting experimental data to a mathematical model. RANSAC has notable advantages over other canopy-fitting approaches as it iteratively determines the best set of points fitting a model within the point cloud. It has successfully been applied to the detection of objects from point clouds (Bretar and Roux 2005; Fontanelli et al. 2007; Forlani et al. 2003, 2006; Reitberger et al. 2007, 2009; Schnabel et al. 2007; Tarsha-Kurdi et al. 2007). We have revised *StarSac* (Shafii et al. 2009), a program was developed using RANSAC and Oliver Kreylos' Virtual Reality Toolkit (VRUI), and verified the results against those of a field survey. Unlike other projects, we used a modified version of RANSAC based on a preliminary maxima filter to find and measure tree canopies.

3. Methods

3.1 Field data collection

The forest consists of four tracts, which were further divided into twenty-one stands ranging in size from four to forty ha. Stand inventory was taken for standing live trees greater than 15.24 cm Diameter at Breast Height (1.37 m) (DBH). The primary objective of the inventory was to estimate total biomass and by extension total forest carbon. The variable plot method outlined in Dilworth and Bell (1963) was used. A Basal Area Factor (BAF) was selected for each stand prior to sampling to produce an average of six to eight “in” trees per plot. Plots were spaced across each tract on a 50m x 100m grid. A total of 660 measure plots were installed.

3.2 Regression models

A regression model of the allometric relationship between height and DBH derived from field data was used to predict DBH from LiDAR derived tree heights. A general non-linear regression model for all species was used to establish DBH from height using measured trees from plots within the same stand. The equation used for regression analysis takes the standard form of:

$$DBH = a * H^b \quad (1)$$

Where H is total tree height; a and b are spatially variable regression coefficients.

3.3 LiDAR data collection

LiDAR data were collected for 1796 ha on March 17th, 2008, conducted with an Optech 3100 sensor mounted in a Cessna Caravan 208B, with specifics shown in Table 1. Instrumentation was set to yield an average native pulse density of ≥ 6 pls/m² over terrestrial surfaces. The TerraScan® software suite was used to classify ground and non-ground points (Soninen 2004).

Table 1: Data Collection statistics.

Sensor	Optech 3100
Survey Altitude (AGL)	900 m
Pulse Rate	> 71 kHz
Pulse Mode	Single
Mirror Scan Rate	52 Hz
Field of View	28° (\pm from nadir)
Overlap	100% (50% Side-lap)

3.4 StarSac

The current RANSAC algorithm is summarized in pseudo-code below, with references to sections that explain key parts of the algorithm in greater detail.

- 1) For all locally maximal LiDAR points Max_i for $i=1 \dots n$: (Section 3.4.1)
 - a) Find set of points around Max_i to create a fixed-size window, one large enough to contain most canopies.
 - b) For iterations $j=1 \dots T$: (Section 3.4.2)
 - i) Randomly select a subset of window points to create model M_j with Max_i as its peak, reject if shape is inappropriate. (Section 3.4.3)
 - ii) Create consensus set C_j for M_j , determine radius. (Section 3.4.4)

- iii) If M_j has at most a (predefined) ratio of outliers to inliers, grade and compare it with the best model M_{best} . Otherwise, ignore it. Keep track of M_{best} . If no previous model was found then M_j is chosen as M_{best} assuming that its ratio of outliers to inliers is appropriate. (Section 3.4.4)
 - iv) If M_{best} found, mark it.
- 2) Visualize canopy-approximating models as shaded surfaces for delineation. (Figure 6)
 - 3) Calculate a height for each canopy-approximating model. (Section 3.4.5)

3.4.1 Local Maxima

Maximum points Max_i are first return points (classified as non-ground) identified during pre-processing. Each maximum is selected based on the fact that it is higher than points inside of a $1.5m \times 1.5m$ window centered on the maximum (see Figure 2). This small box is large enough to capture most peaks in our test data sets.

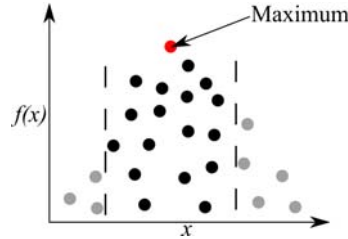


Figure 2: An artificial sketch of a LiDAR point that has been identified as a maximum where each point is defined by the one-dimensional function $f(x)$. The maximum is higher than the other points inside of the box indicated by the dashed lines.

3.4.2 Number of Model Iterations

We calculate the number of RANSAC iterations T according to Schnabel et al. (2007), based on the probability that an appropriate model is found. If a window consists of N points, k points are used to instantiate our model. A good consensus set (i.e., inliers) consists of at least c points. The probability of finding an appropriate model in a single pass is:

$$P(c) = \frac{\binom{c}{k}}{\binom{N}{k}} \approx \left(\frac{c}{N}\right)^k \quad (2)$$

After picking s poor models, the probability of detecting an appropriate model is calculated by evaluating $P(c,s)$:

$$P(c,s) = 1 - (1 - P(c))^s \quad (3)$$

If we were to solve for s , we can calculate the number of candidates T required to detect shapes of size c with probability $P(c,T) \geq p_i$ as:

$$T \geq \frac{\ln(1 - p_t)}{\ln(1 - P(c))} \quad (4)$$

The denominator of (44) can be approximated by the Taylor series $\ln(1 - P(c)) = -P(c) + O(P(c)^2)$ and (44) can be written as:

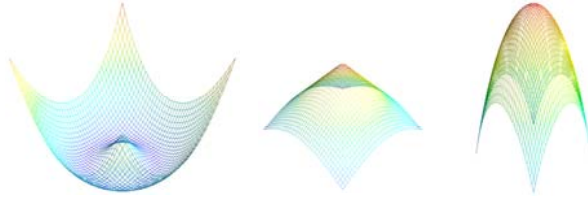
$$T \approx \frac{-\ln(1 - p_t)}{P(c)} \quad (5)$$

We use (55) to define our number of iterations per window.

3.4.3 Model Creation

Our algorithm is currently capable of fitting a “shape-shifter” to data points. Each shape has a parameter defining its central peak (x_c, y_c, z_c) (defined by Max_i used to create the current window) and one parameter α which interpolates between a cone and a paraboloid. If α is zero, the shape is similar to a cone; if it is one, it resembles a paraboloid (Figure 3).

$$f(x, y) = -\alpha * ((x - x_c)^2 + (y - y_c)^2) + (\alpha - 1.0) * \sqrt{((x - x_c)^2 + (y - y_c)^2)} + z_c \quad (8)$$



(a) Shape-shifter with $\alpha=-1.0$. (b) Shape-shifter with $\alpha=0.0$. (c) Shape-shifter with $\alpha=1.0$.

Figure 3: Renderings of the shape-shifter (88) using various values of α . The valid range for α is 0-1.0, as an α value of -1.0 creates an unusual shape as shown in Figure 3(a).

3.4.4 Consensus Set and Model Grading

Each model’s consensus set C_j is created by selecting inlier points within the window. We assemble C_j by computing the $f(x, y)$ value for each window point and comparing that value with the point’s z coordinate. If the difference is smaller than a pre-defined error metric ϵ , the point is added to C_j . From C_j , one can then calculate the radius by calculating the two-dimensional, (x, y) distance between the model’s central peak and the inlier point furthest from the peak. In order to compare against other models, each model is graded based on the number of inliers. The model with the best grade (most inliers) is chosen.

3.4.5 Tree height

To calculate the height for each tree, we use Hardy’s (1971) multiquadric method to reconstruct the ground surface beneath the tree crown. We then subtract the elevation of the hardy surface at the (x, y) location of the crown apex from the height (z -coordinate) of the crown apex.

3.5 LiDAR inventory

StarSac was run in a batch process over all LiDAR data in stands where field data was collected. The classified LAS files were subset into blocks containing $\approx 150,000$ points. Each block was further subset into ground-only points and points classified as first return and vegetation. LiDAR data was processed using the LibLAS (Loskot 2008) command line tools and application programming interface (API). *StarSac* output was collected in a PostgreSQL (The PostgreSQL Global Development Group 2005) relational database with the PostGIS (Holl and Plum 2009) spatial object extensions.

To test the accuracy of the tree identification method outlined above, “in” trees were identified from the LiDAR-derived trees based upon DBH, BAF, and distance to the nearest plot center used in the field inventory. Stand-specific, non-linear regression coefficients derived from field data were used to regress LiDAR-derived tree height to tree DBH. Once DBH was modeled, the status of all trees with regard to the BAF was assessed based upon the distance between the tree and the plot center.

4. Results

4.1 Field inventory

The BAF selected for each plot resulted in between 5 and 8 trees per plot. Basal area ranges between $36 \text{ m}^2\text{ha}^{-1}$ to $74 \text{ m}^2\text{ha}^{-1}$ and generally varies with DBHq though with greater magnitude.

4.1.1 Regression models

The LiDAR methods (*StarSac*) employed here do not differentiate between species, thus regression coefficients were developed using all trees within a given stand. Visual inspection of Figure 4 indicates similarity between species, justifying the application of *StarSac*. Figure 5 represents the results of the regression of DBH and height for all field-measured trees.

4.2 LiDAR analysis

Individual tree identification using the RANSAC method was effective in most cases. Visual inspection of the consensus sets (Figure 6) identified was used extensively in testing the impact

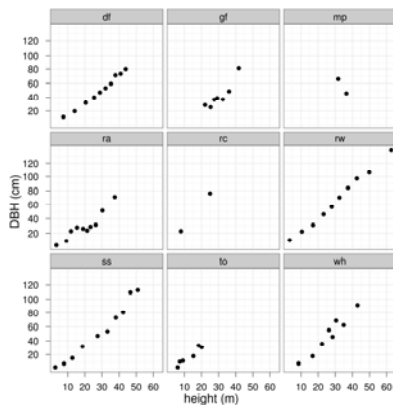


Figure 4: Average DBH within height bins. df, Douglas fir; gf, Grand; mp, big-leaf maple; ra, Red alder; rc, Western red cedar; rw, Coast redwood; ss, Sitka spruce; to, Tan oak; wh, Western hemlock.

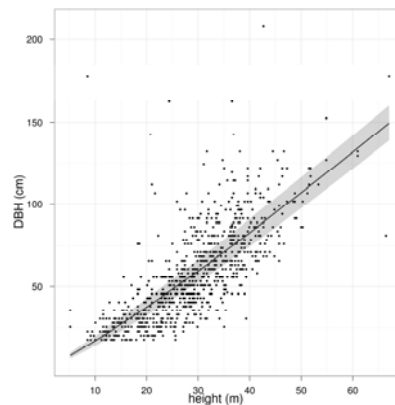
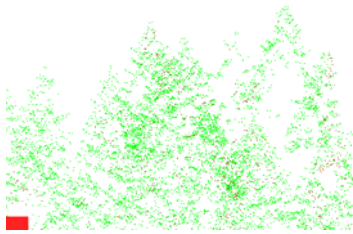


Figure 5: Height-DBH regression for all trees.

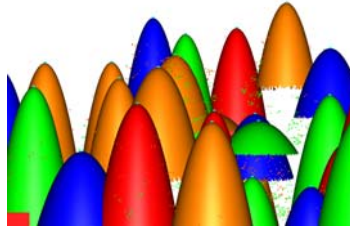
of changes to the approach.



(a) Raw LiDAR points.



(b) Raw LiDAR points with consensus sets and maximum points.



(c) Raw LiDAR points with rendered graphs.

Figure 6: An example of three-dimensional graphs that depict the models used to identify the canopies in StarSac. Each graph is created by using the model's equation and radius. The raw LiDAR rendering is showing in Figure 6(a). In Figure 6(b), central peak points are rendered as thick, cyan points and consensus set points are colored red. The resulting graphs of the canopies are shown in Figure 6(c).

4.3 LiDAR inventory

The LiDAR inventory method was compared with field methods for the determination of basal area. Stand-aggregated basal area estimates were derived using the basal area calculated for each plot. Figures 7 and 8 show the basal area and tree count comparison between methods.

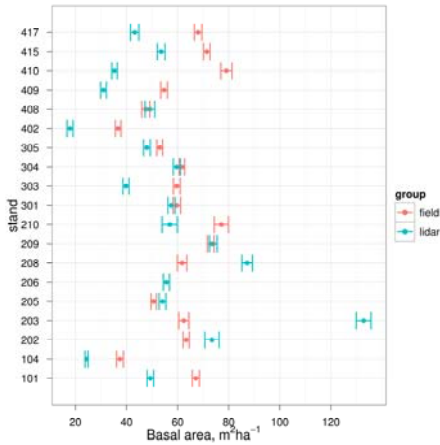


Figure 7: Average plot basal area comparison between LiDAR and field methods with 95% confidence interval whiskers.

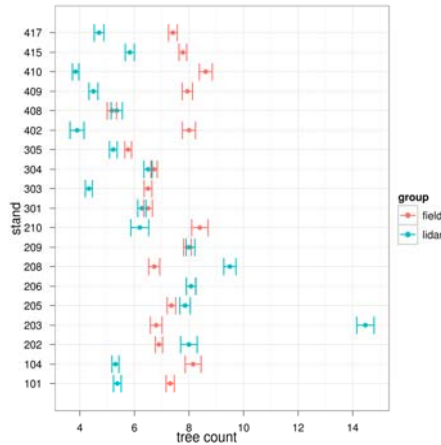


Figure 8: Average plot tree count comparison between LiDAR and field methods with 95% confidence interval whiskers.

Figure 9 indicates that the LiDAR method results in greater basal area estimation in the 50 cm to 125 cm DBH range while field methods estimate greater basal area in the 12 cm to 50 cm and 150 cm to 350 cm ranges.

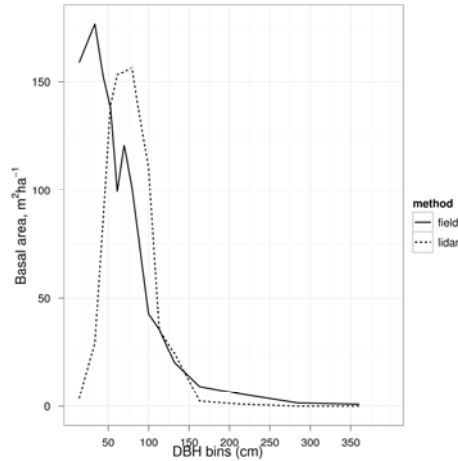


Figure 9: Density plot of basal area by DBH histogram bins for LiDAR and field methods.

The results show strong agreement in many stands while a few stands show some disparity. In general, field methods resulted in a higher tree count and greater basal area.

Table 2 shows tree count/plot averaged across all plots. While there are significant differences in the count and basal area between the LiDAR and field methods for given stands, the combined statistics indicate that the results of field and LiDAR methods are within a RMSD of less than $3.5 \text{ m}^2\text{ha}^{-1}$ (see Table 3).

Table 2: Average plot tree count and basal area for all plots.

LiDAR count	Field count	(average) LiDAR basal area, m^2ha^{-1}	(average) Field basal area, m^2ha^{-1}
6.64	7.17	57.07	60.33

Table 3: Paired t test and RMSD for tree count and basal area for all plots.

df	Tree count t	Tree count P	BA ^a t	BA ^a P	Count RMSD	BA ^a RMSD
301	-1.807	0.072	-1.278	0.202	0.530	3.261

^a basal area

Paired t test between LiDAR and field methods indicate that the means for all paired plots were not significantly different for tree counts ($P=0.072$) or basal area ($P=0.202$).

4.4 Factors affecting agreement between LiDAR and field measures

While overall differences between the methods are not significant, there is obviously some variation between the means for LiDAR and field methods for both tree count and basal area (Figures 7 and 8). As the van Eck forest stands were classified by canopy density and DBH classification, we can assess the impact of these generalized stand characteristics on the differences between the two measurement methods. This is accomplished using ANalysis Of

Variance (ANOVA) for the regression of the stand characteristics (*predictor*) and the log-transformed ratio of basal area estimates (*response*) from the two basal area estimation methods

A two-way ANOVA reveals that neither the interaction of canopy cover and DBH, or DBH alone has significant effects, but that canopy cover alone has an impact upon the variation between measures (Table 4). A one-way ANOVA (Table 5) reveals that the variance in estimated basal area between methods is significant in stands classified in the 40-60% canopy coverage range. Levene’s test reveals that the variance is homogeneous between canopy cover levels (Table 6), validating the assumptions in the one-way test.

Table 4: Two-way ANOVA test for the influence of canopy cover and DBH classification on variance between estimation methods.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
dbh class (dbh)	1	0.00	0.00	0.00	0.9972
canopy density class (dens)	1	5.81	5.81	8.59	0.0036
dbh:dens	1	0.95	0.95	1.41	0.2361
Residuals	298	201.39	0.68		

Table 5: One-way ANOVA test for the influence of canopy cover on variance between estimation methods.

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-0.1984	0.1390	-1.43	0.1547
40-60% cover	-0.7076	0.2949	-2.40	0.0170
60-80% cover	0.0702	0.1482	0.47	0.6359

Table 6: Levene’s test of one-way ANOVA residuals from the influence of canopy cover on differences between basal area measures.

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	0.6413	0.0879	7.29	0.0000
40-60% cover	-0.0336	0.1865	-0.18	0.8571
60-80% cover	-0.0074	0.0937	-0.08	0.9375

Figure 10 shows the effect of canopy density on tree count and basal area estimation by LiDAR and Field methods. Mid-density stands (40%-60%) show significant variation in basal area estimates. Diameter classes did not strongly influence differences in tree counts or basal area estimation by the two methods (Table 4). Figure 11 shows the variability between methods across the range of DBH classes and suggests that variation between basal area methods is greatest in the 0 cm to 20 cm DBH and 61 cm to 81 cm DBH classes.

5. Conclusions, Future Research

This research gives insight into forest inventory from aerial LiDAR data across forest stands that are heterogeneous with regard to management history, species mix, and site characteristics. Tree detection and height estimation is accomplished without the use of regression models or gridded data. The replication of a variable plot method was used so that results of field and RANSAC-based LiDAR methods are comparable. The comparisons in Section 4 indicate that the tree identification and delineation-based LiDAR inventory method, applied to dense, mixed-species stands on variable terrain,

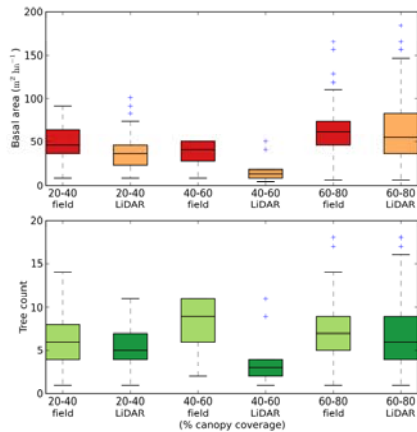


Figure 10: Variation between LiDAR and field based tree count and basal area estimates by percent canopy cover classes. Box extends from the lower to upper quartile values of the data, with a line at the median.

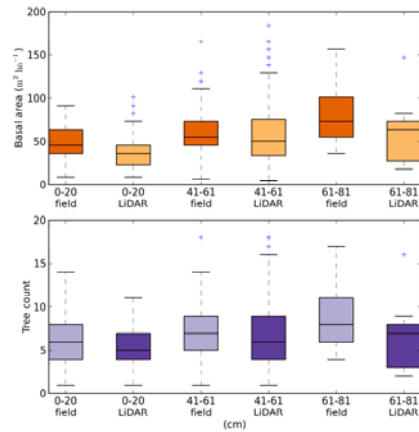


Figure 11: Variation between LiDAR and field based tree count and basal area estimates by diameter classification. Box extends from the lower to upper quartile values of the data, with a line at the median.

yields similar tree count and basal area estimates to the field inventory method. The small difference between average basal area estimated by the two methods indicates that though some trees are not identified using the LiDAR method, they tend to be smaller trees and contribute less to the aggregate statistics.

There exist multiple directions for future research. We intend on refining the RANSAC method such that LM filtering may be eliminated from the algorithm and the sub-canopy vegetation characteristics can be assessed as well. Additionally, we will investigate ways to incorporate methods identified in (Schnabel et al. 2007) for preliminary selection of model parameters using point normals. We hope to improve *StarSac* so that it may provide information about a range of structural measures relating to habitat, fire behavior, and forest health.

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